



Credit risk and its systemic effects Uruguay Case

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Credit risk and its systemic effects^{*}

Victoria Landaberry, Fabio Caccioli, Anahi Rodriguez-Martinez, Andrea Baron, Serafin Martinez-Jaramillo and Rodrigo Lluberas

Abstract

Interconnectedness among financial institutions has been recognized as one of the most important factors for the amplification of the Global Financial Crisis (GFC). Since then, many different research approaches have been developed in order to study systemic risk and its relationship with interconnectedness. In this work, we propose to use a systemic risk metric for an extended network which includes the interbank network, the banks-firms bipartite network and the intra-firm exposures network in Uruguay. This is one of the first works, to the best of our knowledge, in which the intra-firm exposures network is estimated with such an accuracy by using information from a firm survey and is used for the computation of a systemic risk metric. Given that the survey only asks for the three most relevant debtors and creditors, we have to complete the full intra-firm exposures matrix by resorting to two well-known methods: the Maximum entropy and the Minimum Density; additionally, we use an additional method which takes into account the known entries of the matrix obtained from the survey. Our results show an important underestimation of systemic risk if the information of intra-firm exposures is ignored. Moreover, even if the marginal liabilities or assets are used as an indicator of systemic importance for firms is used, important network effects are ignored. The paper has several contributions among which the most important one is the precise estimation of the contribution of intra-firm exposures to the overall systemic risk.

1 Introduction

The increasingly complex and interrelated connections in the financial system are considered to be one of the main sources of risk amplification and propagation of shocks. This was made evident in the worst possible way during the GFC after the fall of Lehman Brothers.



Central Banking



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Contribution

- We use a systemic risk metric for an extended network which includes the interbank network, the banks-firms bipartite network and the intra-firm exposures network in Uruguay.
- This is one of the first works, to the best of our knowledge, in which the intra-firm exposures network is estimated with such an accuracy by using information from a firm survey and is used for the computation of a systemic risk metric.
- The main contribution of the paper is the precise estimation of the contribution of intrafirm exposures to the overall systemic risk.
- Our results show an important underestimation of systemic risk if the information of intra-firm exposures is ignored. Even if the marginal liabilities or assets are used as an indicator of systemic importance for firms, important network effects are ignored.



Motivation

Motivation

- The increasingly complex and interrelated connections in the financial system are considered to be one of the main sources of risk amplification and propagation of shocks. These interconnections among financial entities have been modelled by resorting to network theory and models.
- Nevertheless, contagion through commercial indebtedness among firms or economic sectors has had less attention, Acemoglu et al. (2016), mainly due to the lack of information.
- Currently, it is possible to find some works that include the real sector of the economy and its relationship with the banking system: Poledna et al. (2018) and T. C. Silva et al. (2018).
- This work aims to contribute in filling this gap by building a commercial and financial debt network for Uruguay.



- 1. We use **firm level survey conducted to 240 Uruguayan firms** by the Central Bank of Uruguay in **October 2018** which contains information for the three most relevant debtors and creditors, the total amount lent or borrowed and the total number of creditors and debtors.
 - The Central Bank of Uruguay conducts a survey on commercial debt to a representative sample of firms with more than 50 employees.
 - The sample excludes firms belonging to the primary activity sector, financial intermediation, the public sector or real state activities.
- 2. A second database contains **balance sheet information for 2015**. A larger sample of the commercial credit survey, this survey is representative of firms with more than 10 employees.
 - We use the Consumer Price Index to update balance sheet information until October 2018.



- 3. Another important data set is the **Central Bank Credit Registry database** containing all the loans given to firms by banks; this data set allows us to identify all the credit lent by financial institutions to companies and construct the bank-firm network.
- We combine the information obtained from the survey with balance sheet and credit registry data to build a commercial and financial debt network.

Available data	
Banks	11
Other financial institutions	15
Survey firms	240
Survey firms + main creditors + main debtors	1073
Firms with bank credit	613

Table 1: Banks-Firms Network



In particular, we obtain three networks:

- I. Firm-Bank network: 11 banks and 1073 firms (1 bank only provides mortgage credit to families).
- **II. Financial institutions network:** 26 institutions (11 banks and 15 other financial institutions).
- III. Firm-Firm network: 1073 firms.

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Table 1: Banks-Firms Network



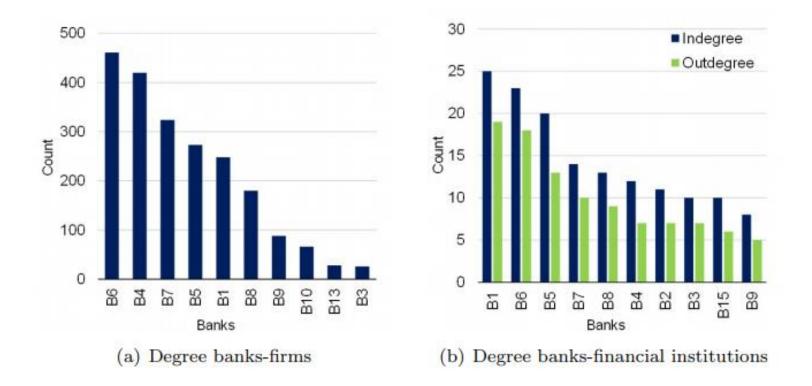


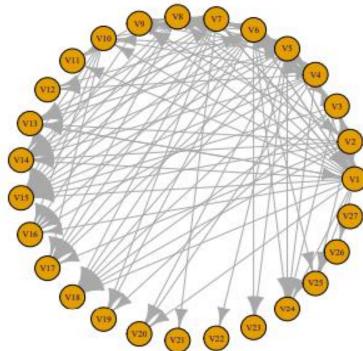
Figure 1: Financial institutions' degree

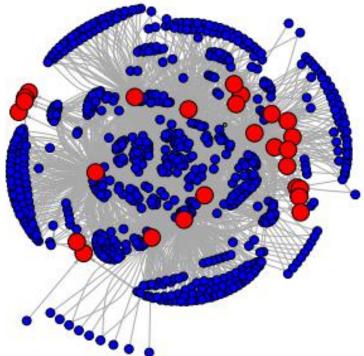


Methodology

Methodology. Network metrics

- In order to characterize the network and identify the nodes (banks) that are more central we use conventional measures of centrality.
- Figure (a) shows the network representation of the inter-bank exposures network while Figure (b) shows the bank-firm exposures, being the red nodes the banks and the blue nodes the firms.







Methodology. Beyond inter-bank exposures

- In Poledna et al. (2018), the authors characterize a useful meta exposures matrix, the different exposures which link the banking system with the real economy, represented by the firms that borrow from the banking system.
- There are links between banks (inter-bank), links between banks and firms (firms deposits at banks and banks credits to firms), and links between firms (intra-firm). This can be represented by a matrix with the following block structure:

$$W_{nxn} = \begin{bmatrix} BB_{bxb} & BF_{bxf} \\ FB_{fxb} & FF_{fxf} \end{bmatrix}$$

where *BB* is the inter-bank exposures matrix, *BF* is the bank-firms loans matrix, *FB* is the firms' deposits at banks and *FF* is the intra-firm exposures matrix.



Methodology. Beyond inter-bank exposures

- Then, we perform a systemic risk analysis of the whole system represented by the matrix W_{nxn} . To this end, we use an extension of the DebtRank algorithm that also accounts for the bank-firm and firm-firm interactions.
- Given the equity of institution *i*, E_i , the relative loss of equity h_i of institution *i* and the exposures of institution *i* to institution *j*, $(W_{nxn})_{ij}$
- The algorithm is the following:

$$h_i(t) = \min\left\{1, \sum_j \frac{(W_{n \times n})_{ij}}{E_i}h_j(t-1) + h_i(1)\right\},\$$

where $h_i(0) = 0$ and $h_i(1)$ is the relative loss of *i* associated with an external shock.

The DebtRank can be used to compute the impact and the vulnerability for each institution. The impact is the relative loss that the failure of an institution would cause to the system. The vulnerability is the average relative loss from the defaults of all the other institutions.



Methodology. Network reconstruction methods

- We consider alternative methods to **reconstruct the firm-to-firm network**:
 - I. Maximum Entropy (ME), Upper and Worms (2004);
 - II. Minimum Density (MD), Anand et al. (2014); and
 - III. Fitness model, Caldarelli et al. (2002); Park and Newman (2004); Squartini and Garlaschelli (2011).
- ME tends to create complete networks in which all entries are as homogeneous as possible while being compatible with the constraints provided by the total borrowing and lending of each individual institution.
- MD allocates the total amount lent to and borrowed from each bank while using as few links as possible, thus producing a very sparse network which represents a lower bound in terms of connectivity.



Methodology. Network reconstruction methods

- We also use a combination of a fitness model and maximum entropy. The fitness model can in fact be used to compute probabilities for links that are known to exist in the network.
- These probabilities are computed such that, on average, the number of creditors and debtors of each individual institution is equal to the one observed empirically.
- The linking probabilities can then be used to produce adjacency matrices that correspond to plausible network structures compatible with the number of counterparties of each institution.
- RAS algorithm can then be used to assign weights to the existing links. This method generates networks with a connectivity degree that is intermediate between those of the ME and MD. These and other methods are well documented in Anand et al. (2018).



Methodology. Reconstruction of firm-to-firm network

- We have a system of *N* firms. For each firm *i* we know the total amount a_i of loans to other firms, the total amount l_i of money borrowed from other firms, the number of creditors k_i^{out} and the number of debtors k_i^{in} (the convention we use is that a link goes from the borrower to the lender).
- We also know the identity of a subset of creditors and debtors. We denote these subsets respectively by v_i^{out} and v_iⁱⁿ.
- We have an incomplete matrix of intrafirm exposures, which we need to fill by satisfying the constraints on the total in and out degree and in and out strength of each node (in and out strengths are a_i and l_i respectively).
- We proceed with a two-step method: first, we reconstruct a binary adjacency matrix that satisfies (on average) the constraint on in and out degree using a fitness model. Second, we assign weights to the links using the RAS method.



Reconstruction of firm-to-firm network. Fitness model

 According to the fitness model the probability that a link from node *i* to node *j* exists is given by (Park & Newman, 2004; Squartini & Garlaschelli, 2011):

$$p_{i \to j} = \frac{x_i^{\text{out}} x_j^{\text{in}}}{1 + x_i^{\text{out}} x_j^{\text{in}}},$$

where the set of variables x_i^{out} and x_i^{in} are called fitness, have to be computed in such a way that the constraints on the in and out degrees are satisfied, i.e. by solving the following set of equations:

$$\begin{aligned} k_i^{\text{out}} - |\nu_i^{\text{out}}| &= \sum_{j \notin \nu_i^{\text{out}}} \frac{x_i^{\text{out}} x_j^{\text{in}}}{1 + x_i^{\text{out}} x_j^{\text{in}}} \quad \forall i \in \{1 \dots N\} \\ k_i^{\text{in}} - |\nu_i^{\text{in}}| &= \sum_{j \notin \nu_i^{\text{in}}} \frac{x_i^{\text{in}} x_j^{\text{out}}}{1 + x_i^{\text{in}} x_j^{\text{out}}} \quad \forall i \in \{1 \dots N\}, \end{aligned}$$

where the set of variables $|v_i^{in}|$ and $|v_i^{out}|$ represent the number of elements in sets v_i^{in} and v_i^{out} (these are the numbers of creditors and debtors for which we know the exposures).



Reconstruction of firm-to-firm network. Fitness model

Once we have solved the above set of equations to determine the values of the x_i 's, we can generate an instance of a binary adjacency matrix by drawing each link $i \rightarrow j$ with probability $p_i \rightarrow j_i$.

Ras algorithm

• Once we have determined which links are present in the network, we have to assign weights to those links. We know the weight of the links from the data (i.e. those in the sets v_i^{in} and v_i^{out}) and thus we assign them the known weights.



Reconstruction of firm-to-firm network. Ras algorithm

To the other links we assign weights through the following iterative procedure (n denotes the iteration):

$$\begin{split} W_{i \to j}^{(2n)} &= \frac{W_{i \to j}^{(2n-1)}}{\sum_{j \notin \nu_i^{\text{out}}} W_{i \to j}^{(2n-1)}} \left(\ell_i - \sum_{j \in \nu_i^{\text{out}}} W_{i \to j} \right). \\ W_{j \to i}^{(2n+1)} &= \frac{W_{j \to i}^{(2n)}}{\sum_{j \notin \nu_i^{\text{in}}} W_{j \to i}^{(2n)}} \left(a_i - \sum_{j \in \nu_i^{\text{in}}} W_{j \to i} \right). \end{split}$$

In even (odd) steps we re-scale the unknown weights such that the sum of the elements in each row (column) is equal to the total amount of debt (credit). Clearly when we enforce the sum over the rows, the one over the columns will be wrong, and the other way around. We iterate the equations until we reach a given precision.



Building a network of effective exposures of banks towards firms

- Let us denote by V_{ai} the exposure of bank *a* towards firm *i*. This is associated in our case with a loan from the bank to the firm. However, in the presence of credit relationships between firms, a bank can be exposed to firms it did not directly lent to.
- If bank a lends to firm i and not to firm j, but firm i lends to firm j, the inability of firm j to pay its debt to firm i may affect bank a.
- In the figure of the left, bank a is only exposed to firm 1, as firm 1 is not linked, through commercial credit, to any other firm. In the right figure, bank a is directly linked to firm 1 and indirectly exposed to firm 3 and firm 4 because they owe to firm 1.

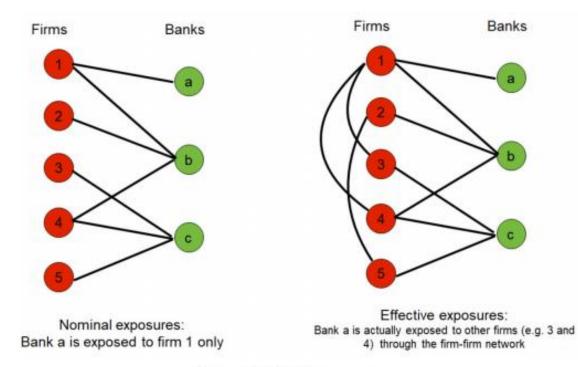


Figure 3: Effective exposures



Building a network of effective exposures of banks towards firms

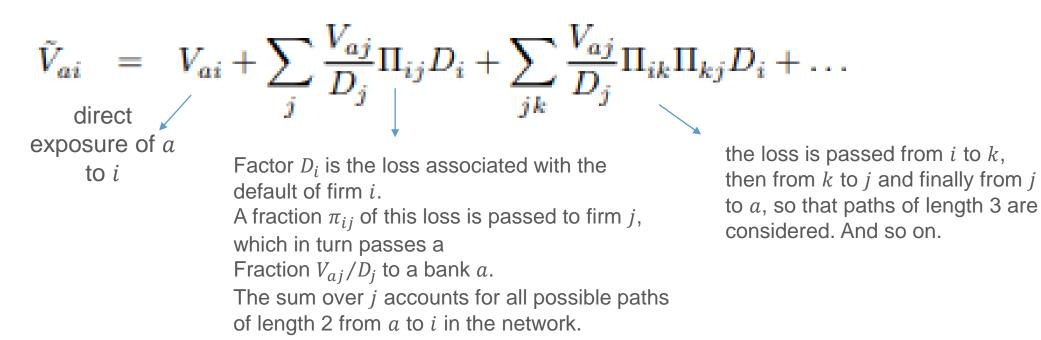
- Rather than trying to construct a micro-founded model of how these type of shocks propagate in the network of firms, we consider the existence of effective exposures of banks towards firms.
- We can say that bank *a* is effectively exposed to firm *i* by an amount equal to:

$$\begin{split} \tilde{V}_{ai} &= V_{ai} + \sum_{j} \frac{V_{aj}}{D_{j}} \Pi_{ij} D_{i} + \sum_{jk} \frac{V_{aj}}{D_{j}} \Pi_{ik} \Pi_{kj} D_{i} + \dots \\ &= \sum_{j} \left[(\mathbb{1} - \Pi)^{-1} \right]_{ij} \frac{V_{aj}}{D_{j}} D_{i}, \end{split}$$

Where firm *j* owns a fraction
$$\pi_{ij} = \frac{W_{ij}}{D_i}$$
 of *i*'s debt.



Building a network of effective exposures of banks towards firms



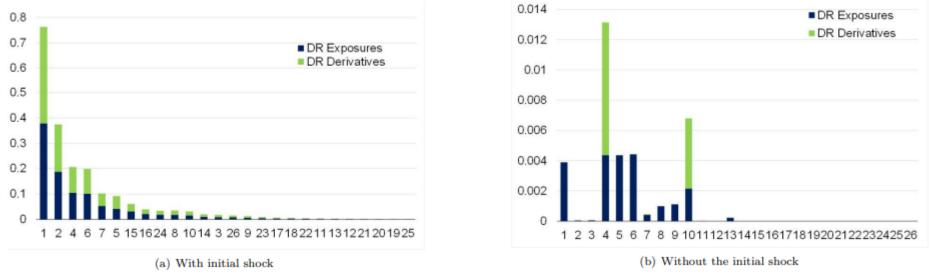
- The underlying assumption is that if a firm defaults, its creditors (linearly) propagate some loss to their creditors, and so on. In practice, the propagation could stop if some creditors absorb the loss without passing it further on.
- The exposures calculated in the above Equation are therefore only an upper bound to effective exposures, while nominal exposures are a lower bound.



Results

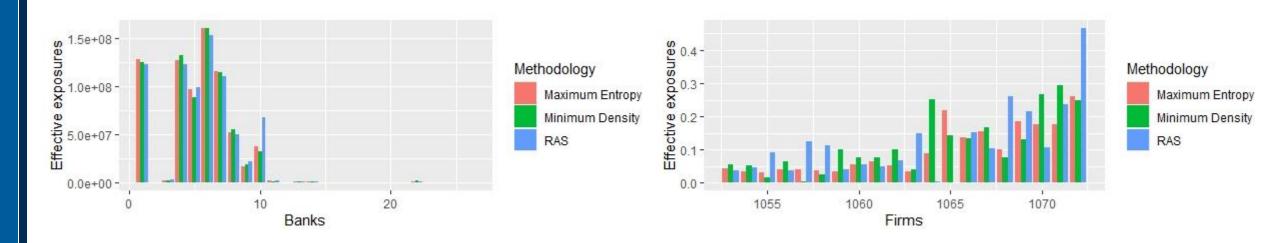
Results. DebtRank algorithm to the interbank exposures network.

- This network consists of two layers: the unsecured lending layer and the derivatives layer.
- In these Figures, we present the systemic risk profile for the banking system in Uruguay. Neglecting the derivatives layer can underestimate bank systemic importance in the network.



Results. Nominal and effective exposures.

- In the figure of the left, we show the effective exposure effect of the inter-bank network. Each entity has three bars of effective exposures (one bar for each methodology). The graph shows that the effect is similar for each methodology, but for some entities the effective exposure is slightly larger in the RAS methodology, for example entities 5 and 10.
- In the right Figure, we show the difference of effective exposure's effect of each methodology for the intra-firm network.

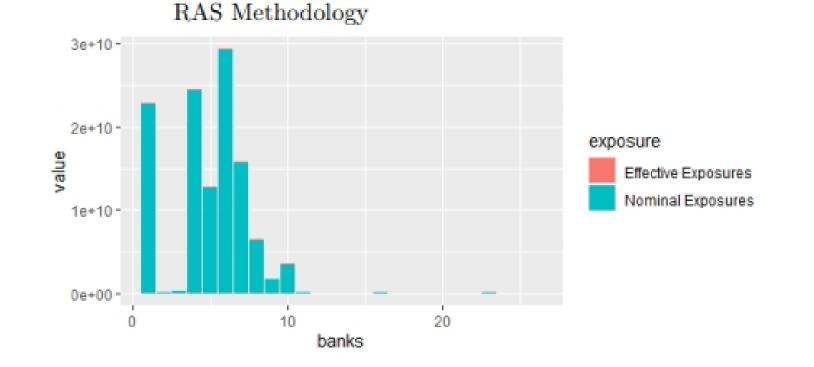




Results. Nominal and effective exposures.

- We show an aggregated view for the nominal an effective exposures in the inter-bank network for the RAS Methodology.
- We find that the effective exposures are not much larger than the nominal ones for this system, however it is important to take into account this effect and in order to not underestimate the inter-bank network and its effects on systemic risk.

Effective vs Nominal Exposures



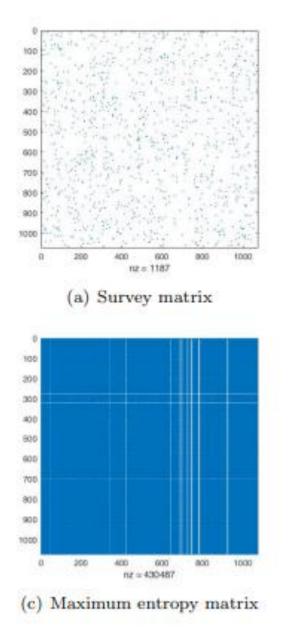


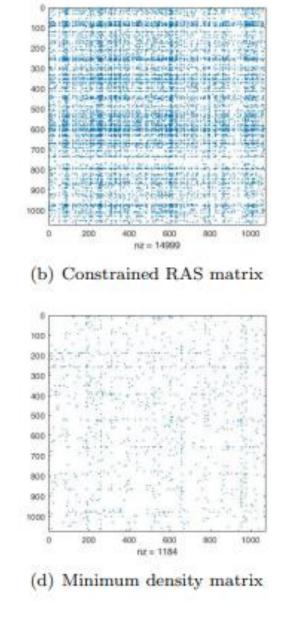
Results. Matrix of exposures estimation methods.

- The information from the survey gives us data for 1,187 observations (Figure a) which presents a sparse matrix; when we apply the RAS algorithm to complete the matrix from the known information (Figure b) we increase the non zero elements to 14,999 observations.
- This means that the RAS algorithm gives us more useful information for vulnerability and impact analysis. In Figure c, we use the Maximum Entropy methodology which shows a plenty matrix with 430,487 observations.
- On the other hand, with minimum density, the number of observations decreases to 1,184 almost the same number of observations from the survey (Figure d).
- In the right Figure, we show the intersection between the survey observations (blue) and Anand's Minimum Density methodology (red). We find only four intersections between the two matrices (red circles).

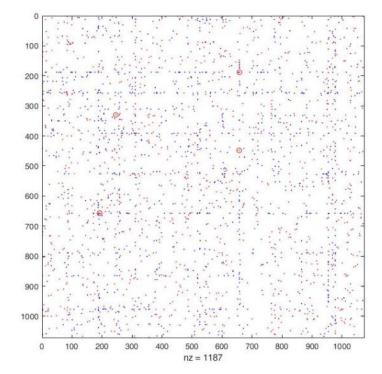


Results. Matrix of exposures estimation



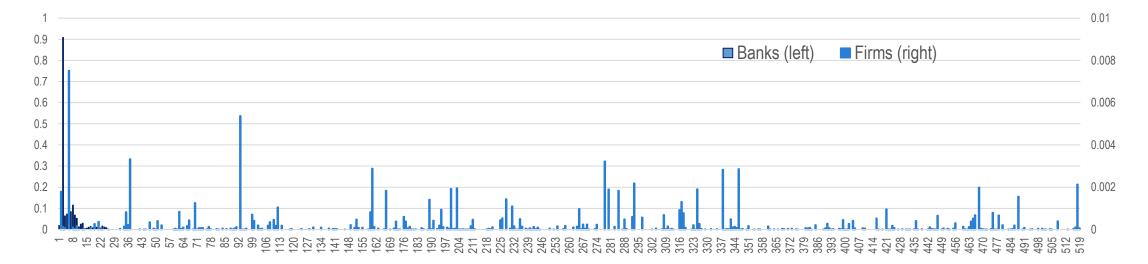


Intersection Survey and Anand matrix



Results. Intra-firm exposures: banks and firms vulnerability. Base case

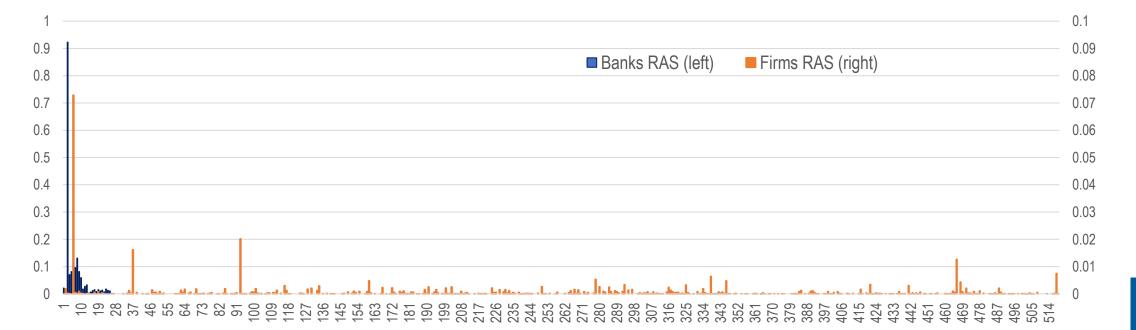
- We found that at least one bank is importantly vulnerable when we include intra-firm exposures information, its vulnerability reaches 90%, an important effect on the interbank network.
- It is worth noting that this is a small, non-systemic bank with a very small proportion of total credit. Concerning firms, we find that in some cases the vulnerability goes from 0.1% to 0.8%.





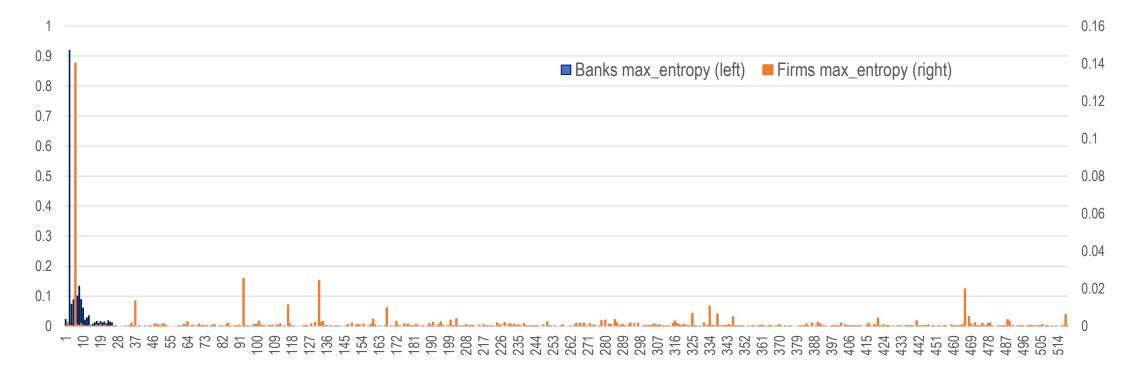
Results. Intra-firm exposures: banks and firms vulnerability. RAS method

- If we reconstruct the intra-firm matrix using the RAS methodology, we find a slightly increase in the vulnerability of some banks. The same applies to firms, whose vulnerability goes from 1% to 7%.
- At the aggregate level, the vulnerability of firms when we consider intra-firm exposures is around 21% higher compared to the base case. The 21% results from computing the aggregated sum of the differences for each firm between the intra-firm network from the RAS methodology and the base case.



Results. Intra-firm exposures: banks and firms vulnerability. Maximum entropy method

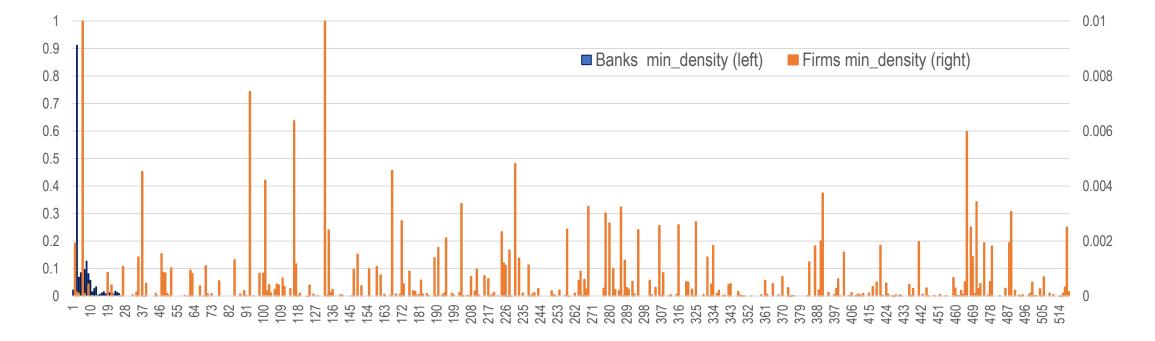
The analysis using the Maximum entropy method shows more firms with higher vulnerability than in the base case and the RAS methodology. The vulnerability of firms goes from 1% to 14%, and the increase in aggregate vulnerability for firms including intra-firm exposures is around 37% relative to the base case.





Results. Intra-firm exposures: banks and firms vulnerability. Minimum density

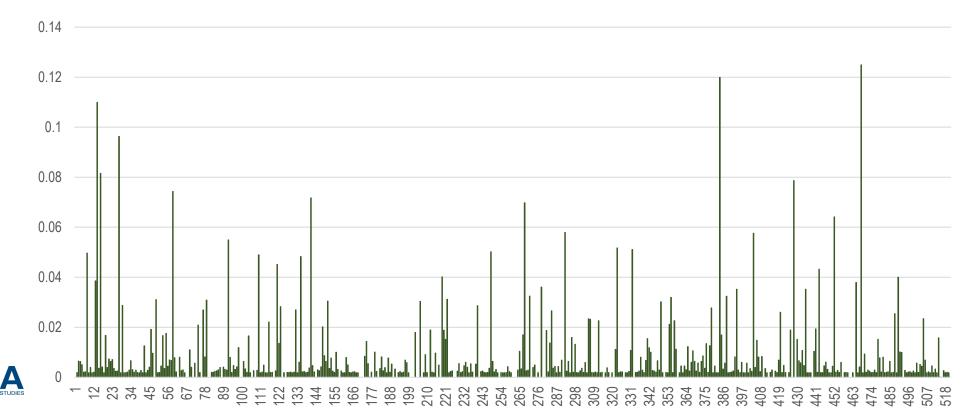
The Minimum density approach shows lower levels of vulnerability for firms, from 0.1% to 1.0% in contrast with Maximum Entropy and the RAS algorithm. Aggregate vulnerability for firms when we consider intra-firm exposures is around 11% higher compared to the base case.





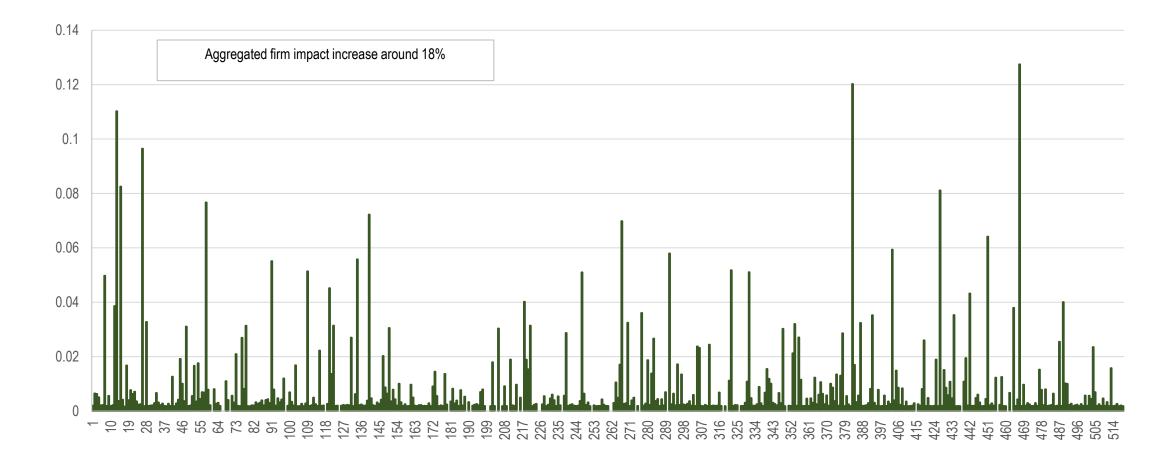
Results. Intra-firm exposures: Firm impact without firm to firm exposures

- In this section, we analyze the changes in firm impact due to the inclusion of intra-firm exposures. We quantify the aggregate firm impact through the different methodologies.
- This Figure shows the case in which the firm impact does not include intra-firm exposures information. Some firms contribute with a larger impact to the overall system, for example entities which show an impact of around 10% and 12%.



Results. Intra-firm exposures: Firm impact. Base case

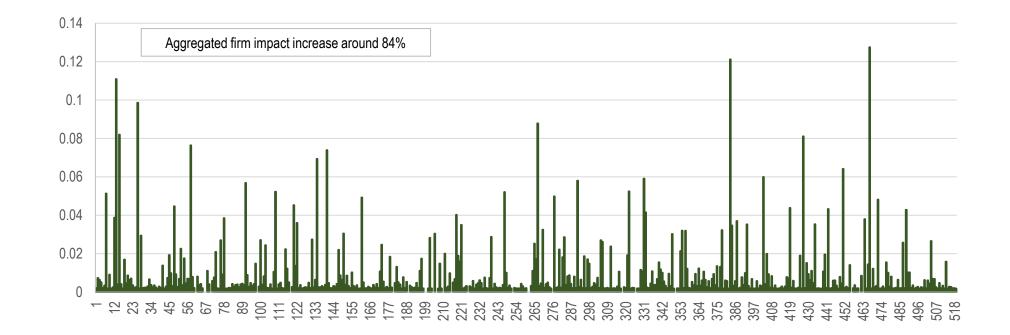
The intrafirm exposures increase aggregated firm impact around 18%.





Results. Intra-firm exposures: Firm impact. Maximum Entropy

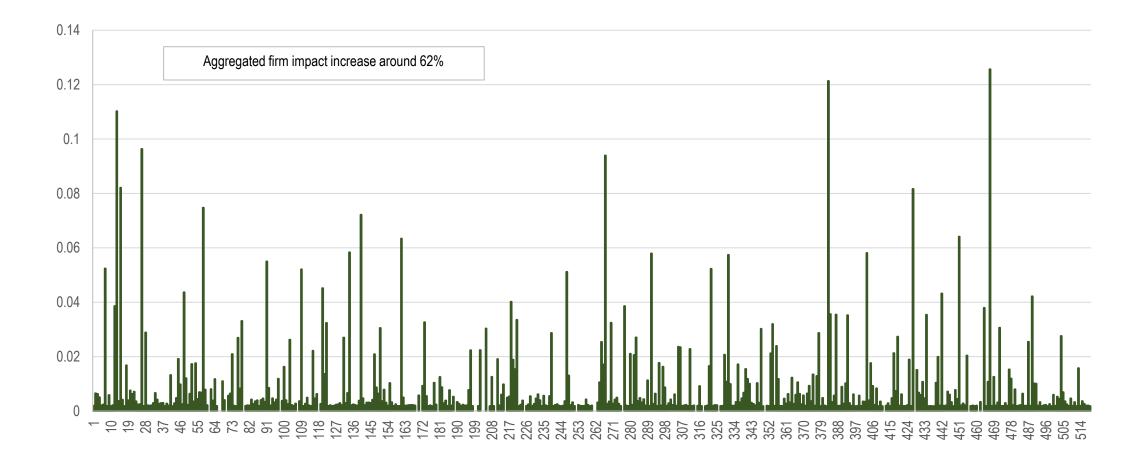
- Intra-firm exposures with the Maximum Entropy show an increase of the aggregate impact, measured by the aggregate losses, of around 84%.
- The 84% results from computing the aggregate sum of the difference between including the intra-firm exposures and without them for the RAS methodology.





Results. Intra-firm exposures: Firm impact. Minimum Density

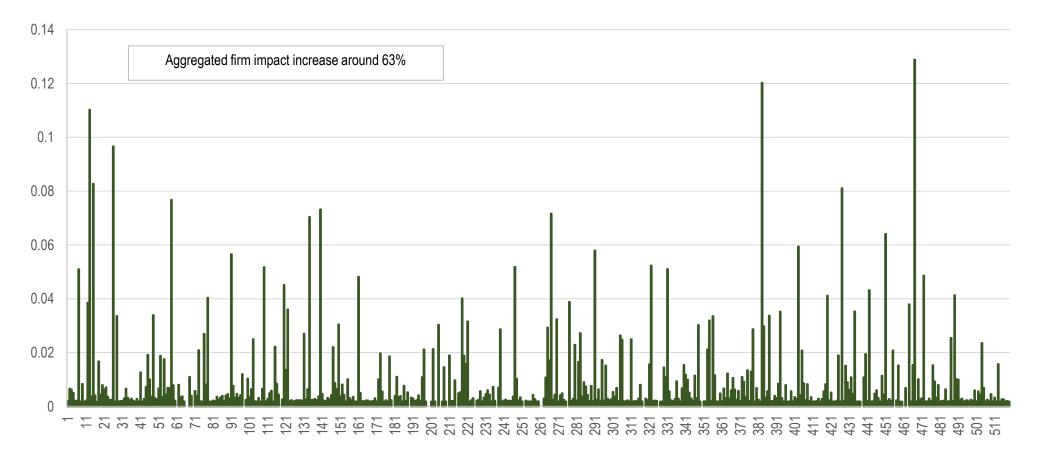
Intra-firm exposures with Minimum Density increase aggregate firm impact by around 62%.





Results. Intra-firm exposures: Firm impact. RAS methodology

Finally, with the RAS methodology, the intra-firm exposures increase aggregate firm impact by around 63%.



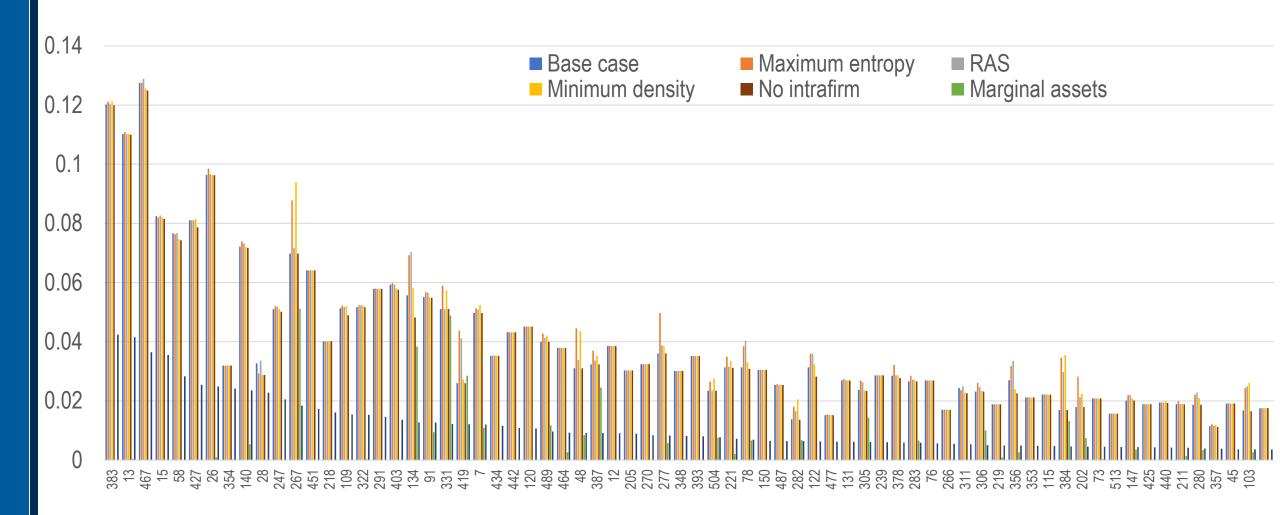


Results. Intra-firm exposures: Ranking by Marginal Liabilities

- We order vulnerability including intra-firm exposures information by marginal liabilities.
- For each entity we have the information for the three different methodologies (RAS, maximum entropy, minimum density), marginal assets, marginal liabilities, vulnerability without intra-firm exposures information (0), and DebtRank (1).
- It is important to highlight that we order the ranking by marginal liabilities because it is the way where the contagion propagates.
- According to RAS and DebtRank methodology most of the entities are not order in the same way that marginal liabilities rank the entities.
- For instance, third firm can affect around 13 percent of equity in firms network.



Results. Intra-firm exposures: Ranking by Marginal Liabilities





Results. Intra-firm exposures: Differences for the top 10 entities according to the different methods.

- We take as pivot the Survey information and we found the that the main differences occur when we order the entities by marginal assets and marginal liabilities, and with maximum entropy and minimum density.
- For the first 10 entities, RAS and Survey information have the same order of entities.

Survey	Max Entropy	RAS	Min Density	Base Case	Marg assets	Marg liab
467	467	467	467	467	267	383
383	383	383	383	383	140	13
13	13	13	13	13	26	467
26	26	26	26	26	13	15
15	267	15	267	15	427	58
427	15	427	15	427	383	427
58	427	58	427	58	467	26
140	58	140	58	140	15	140
267	140	267	140	267	58	267
451	451	451	451	451	451	451

Table 2: Top 10 Ranking table by different methodologies



Conclusions

Conclusions

- The most novel part of this work relies on the estimation of the intrafirm exposures network and its contribution on the systemic risk faced by the banking system. We estimate the intrafirm exposures network by resorting to three alternative methods (MD, ME, RAS).
- We were able to identify systemically important firms on the basis of their impact on banks and other firms taking into account contagion (network) effects.
- The computation of effective exposures show that banks are exposed among them beyond their direct credit lines given to firms through the firm-firm lending relationships.
- If we do not take into account the intra-firm exposures, we will underestimate systemic risk. Moreover, the most important part of the vulnerability of Uruguayan banks to financial contagion comes from the real sector of the economy, in contrast to the well studied interbank exposures.



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